inferred. The other half of the year is shown on another scale of dates, which can be folded down over that represented in the figure.

Supplementing the map there must be given a table of times of sunset for the point X.

To find the time of sunset at any point P for any date, take the paper scale AB (which is a scale of minutes and should be detached from the map), lay the corner A on P, keeping the scale horizontal, and read where it cuts the date-line. This reading gives the number of minutes to be added to the time of sunset at X, which is given in the table.

15. The scheme of Mr. Benson and Mr. Clark may be indicated as follows :—

(*a*) They take Mercator's projection for the map. The latitude scale is a gradually expanding one, and happens therefore to be nearly the same as that we were led to theoretically in what precedes. Mr. Benson and Mr. Clark determined its suitability for straight sunset-lines by experiment.

(*b*) They place the reference point X in the centre of the map, which involves the use of + and − signs, but reduces the size of the map.

(*c*) The sunset-lines are not actually drawn. The dates are indicated at the top and bottom of the map, and a string fastened at X can be stretched to the date so as to form any required sunset-line.

(*d*) No separate scale AB is given. The distance of a place to right or left of the line is to be inferred from the distances between meridians. This is indeed not difficult when the point X is placed in the middle of the map, and the distances are all small.

(*e*) The table giving sunset at X is practically ranged alongside the dates given at the top and bottom of the map, in the form of an accompanying time-scale. This is in many ways convenient, and the only disadvantage is (as indicated in § 11) that the scale is rather small, as four hours are to be represented within the dimensions of the map.

Tables to facilitate the working of Combined Altitudes by Saint-Hilaire's Method. By Lieut. E. B. Simpson-Baikie, R.N.R.

Communicated by the Secretaries.

The most valuable of all methods for finding a ship's position at sea is no doubt the combined altitude method, which consists of either a combination of simultaneous observations of different heavenly bodies, or a combination of successive observations of the same one.

There are several systems of utilising such observations, according to the positions of the bodies. When a star is near the prime vertical, the hour angle and thence the longitude may be calculated from the altitude, declination, and assumed latitude; when near the meridian, the latitude may be found by the "ex-meridian" method.

In Marcq Saint-Hilaire's method the position of the observer is assumed, the altitude of the body calculated for the instant of observation from this assumed position, and compared with the true altitude.

In all methods the combination of the two or more observations is effected by drawing "lines of position" at right angles to the lines of azimuth of the observed bodies through the approximate positions resulting from each observation on a Mercator chart. These "lines of position," being short, may be considered arcs of circles of altitude of the respective bodies, and therefore geometrical "loci" for the "true position," which lies at their intersection.

To ensure accuracy the altitudes should not exceed 80°, and the angular difference (difference in azimuth) between two bodies should not be less than 25° or greater than 155°.

In Saint-Hilaire's method the "lines of position" are displaced as many miles towards (or from) the body as the number of minutes of arc that the true altitudes are greater (or less) than the calculated altitudes.

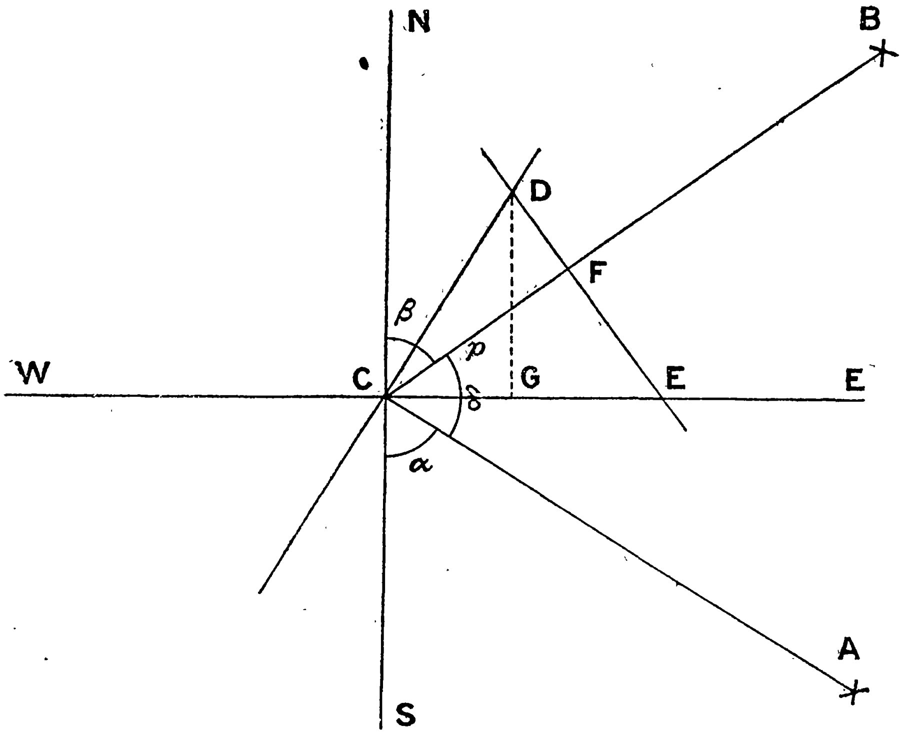
The great advantage of this method over all others is that it may be used with observations of heavenly bodies in any position in azimuth, with only the two restrictions mentioned above, which are common to all methods of combined altitudes. This advantage has been recognised in the Imperial German Navy, where Saint-Hilaire's method (*Höhenmethode*) is now taught almost to the exclusion of all others. In the Royal Navy the junior ranks are instructed in it, and it is a compulsory subject in the course for first-class navigators.

One disadvantage still exists. This is the necessity of obtaining the final result graphically from a chart or sheet of paper—a proceeding which is both clumsy and tiresome. The alternative of calculating the corrections to the assumed position is slightly confusing and somewhat lengthy.

To remove this disadvantage tables have now been computed by which the difference of latitude and difference of longitude between the intersection of the "lines of position" and the assumed position may be found and a final result obtained with few additional figures. These tables are founded on the following considerations, which are explained in "Brent, Walter, and Williams' Ex. Mer. Alt. Tables."

In fig. 1 let NS represent the meridian and WE the parallel of latitude passing through C, the "assumed position." A is one star, whose azimuth is a and whose true and calculated altitudes agree. CD is the line of position due to the observation

of A. B is another star, whose azimuth is β. Let the difference in azimuth between A and B be δ. The difference between the true and calculated altitudes of B is $CF = p$. The



line of position FE, determined by the second observation, cuts the first line of position at D, the "true position."

Now, $$DC = \frac{DG}{\sin DCG}$$

$$\text{I.} \quad \therefore \; DG = DC \cdot \sin DCG,$$

also $$DC = FC \, . \sec DCF.$$

$$\text{II.} \quad DG = FC \, . \sec DCF \sin DCG,$$

but, $$DCG = ACS = \alpha,$$

and $$DCF = (\alpha + \beta) - 90^\circ$$

$$\therefore \sec DCF = \operatorname{cosec} 180^\circ - (\alpha + \beta) = \operatorname{cosec} \delta.$$

Therefore for II. we put : correction lat. $= \dfrac{p \, . \sin \alpha}{\sin \delta}$.

Similarly : $$CG = DC \, . \cos DCG$$
$$CG = FC \, . \sec DCF \, . \cos DCG$$

$$\therefore \quad \text{correction departure} = \frac{p \, . \cos \alpha}{\sin \delta}$$

And, correction longitude $= \dfrac{p \, . \cos \alpha}{\cos \text{lat} \, . \sin \delta}$.

The expressions $\frac{p \,.\, \sin a}{\sin \delta}$ and $\frac{p \,.\, \cos a}{\sin \delta}$ are so related as to make it possible, p being taken as unity, to construct a table with values of a as arguments at the head of the columns and the values of the corresponding complements of a at the foot, each line corresponding to successive values of δ. The table so constructed gives values of a from 0° to 90° for every whole degree, and values of δ from 20° to 70° for every whole degree, and from 70° to 90° for every alternate degree. When δ exceeds 90° its supplement must be used for entering the table.

To prevent error it must be remembered that the arguments for finding the corrections in latitude and longitude due to the difference between the true and calculated altitudes of a star B (see diagram) are the difference in azimuth (δ) between the two stars, and the azimuth (a) of the *other* star, A.

An additional table gives the values of difference of longitude corresponding to departure within suitable limits.